An Evaluation Of Multiwavelet Families For Stereo Correspondence Matching

Pooneh Bagheri Zadeh
Department of Engineering, Faculty of Technology
De Montfort University
Leicester, UK
e-mail: pbz@dmu.ac.uk

Cristian V. Serdean
Department of Engineering, Faculty of Technology
De Montfort University
Leicester, UK
e-mail: cvs@dmu.ac.uk

***Abstract*—This paper presents an evaluation of different types and families of multiwavelets in stereo correspondence matching. Different multiwavelet families with different filter types such as balanced versus unbalanced, symmetric-symmetric versus symmetric-antisymmetric are used. Normalized cross correlation is employed to find the best correspondence points and generate a disparity map. In the case of balanced multiwavelets, due to similar spectral content of the four generated low frequency subbands, they are shuffled to form a single baseband and then this baseband is used to generate a disparity map. However, in the case of unbalanced multiwavelets, the resulting basebands are used to form four disparity maps and then these maps are combined using a Fuzzy algorithm to generate a single disparity map. Middlebury stereo test images are used to generate experimental results. Results show that the unbalanced multiwavelets produce a smoother disparity map with less mismatch errors compared to balanced multiwavelets.**

Keywords-Multiwavelets; stereo correspondence matching; normalized cross correlation;

I. INTRODUCTION

Stereo correspondence is an issue of great importance in the field of computer vision and 3D reconstruction. It concerns the matching of points between a pair of stereo images of the same scene. The disparity is calculated as the distance of the correspondence points when one of the two stereo image pairs is projected onto the other. The disparity map along with the stereo camera parameters are then used to calculate the depth map and produce a 3D view of the scene. Nevertheless, a number of problems such as occlusion, ambiguity, illumination variation and radial distortion limit the accuracy of the disparity map, which is crucial in generating a precise 3D view of the scene [1].

Over the past years much research has been done to improve the performance of correspondence matching techniques. Multiresolution-based stereo matching algorithms have received much attention due to the hierarchical and scale-space localization properties of the wavelets [2][3]. This allows for correspondence matching to be performed on a coarse-to-fine basis, resulting in decreased computational costs. Sarkar and Bansal [3] presented a multiresolution-based correspondence technique using a mutual information algorithm. They showed that the multiresolution technique produces significantly more accurate matching results compared to non-multiresolution based algorithms, at much lower computational cost.

Research has shown that multiwavelets (unlike scalar wavelets) can possess orthogonality (preserving length), symmetry (good performance at the boundaries via linear-phase), and a high approximation order simultaneously [4], which could potentially increase the accuracy of correspondence matching techniques. Bhatti and Nahavandi [5] introduced a multiwavelet based stereo correspondence matching algorithm. They use the wavelet transform modulus maxima to generate a disparity map at the coarsest level. This is then followed by the coarse-to-fine strategy to refine the disparity map up to the finest level. Bagheri Zadeh and Serdean [6] proposed another multiwavelet based stereo correspondence matching technique. They used a global error energy minimization technique to find the best correspondence points between the same multiwavelet's lowest frequency subbands of the stereo pair, followed by a fuzzy algorithm to form a dense disparity map.

In spite of their highly desirable advantages compared to scalar wavelets, the application of different types and families of multiwavelets in stereo correspondence matching has been little investigated in the literature so far.

This paper studies the application of different types and families of multiwavelets in stereo correspondence matching. A multiwavelet is first applied to the input stereo images to decompose them into a number of subbands. Normalized cross correlation is used to generate a disparity map at the coarsest level. In the case of balanced multiwavelets, as the four low frequency subbands have similar spectral content, they are shuffled to generate one baseband, while in the case of unbalanced multiwavelets, the resulting basebands are used to form four disparity maps and then a Fuzzy algorithm is used to combine the four maps and generate one disparity map.

The rest of the paper is organized as it follows. Section II introduces a brief review of the multiwavelet transform. The proposed stereo matching technique for both balanced and

L_1L_1	L_1L_2	L_1H_1	L_1H_2
L_2L_1	L_2L_2	L_2H_1	L_2H_2
H_1L_1	H_1L_2	H_1H_1	H_1H_2
H_2L_1	H_2L_2	H_2H_1	H_2H_2

Figure 1. One level of 2D Multiwavelet decomposition.

unbalanced multiwavelets is discussed in Section III. Experimental results are presented in Section IV and the paper is concluded at Section V.

II. MULTIWAVELET TRANSFORM

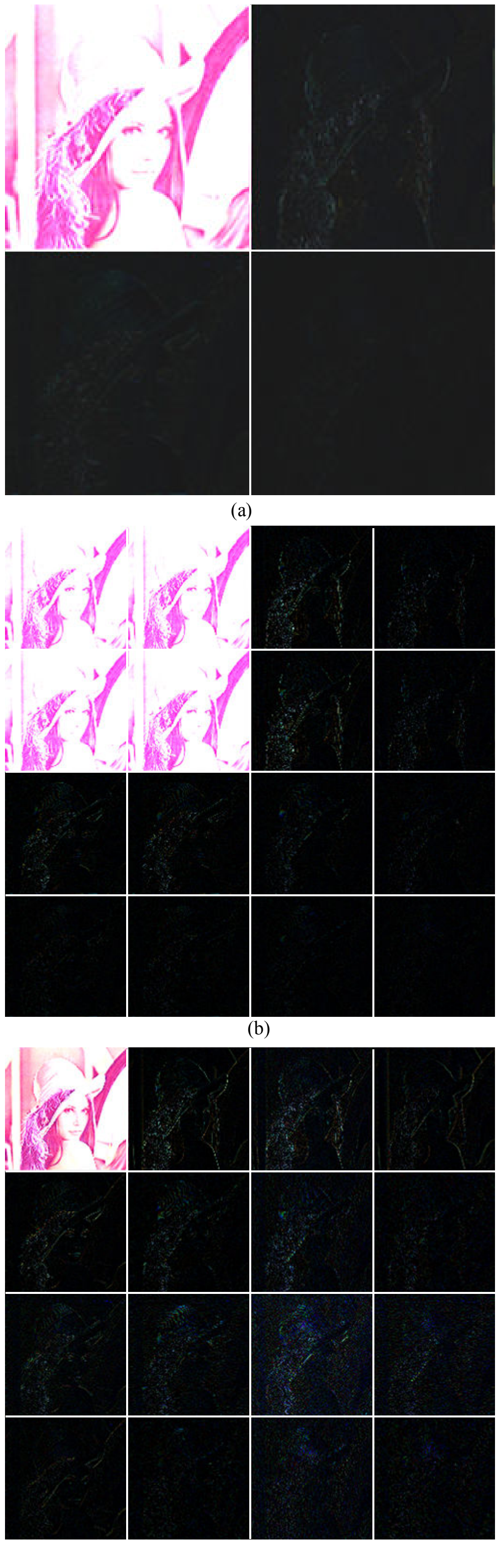
In many respects multiwavelet transforms are very similar to scalar wavelet transforms. In contrast to the wavelet transform, which supports one wavelet and one scaling function, multiwavelets have two or more scaling and wavelet functions. A multiwavelet with two scaling and wavelet functions can be defined as [7]:

$$\begin{aligned}\Phi(t) &= \sqrt{2}\sum_{k=-\infty}^{k=\infty} H_k \Phi(m\,t-k)\\ \Psi(t) &= \sqrt{2}\sum_{k=-\infty}^{k=\infty} G_k \Psi(m\,t-k)\end{aligned} \tag{1}$$

where $\Phi(t)$ and $\Psi(t)$ are the multi-scaling function and multiwavelet function and H_k and G_k are $r\times r$ matrix filters (r is the number of scaling- and wavelet functions). To date, most multiwavelets have $r=2$ [4,7].

One level of decomposition for a 2D multiwavelet with multiplicity 2 produces sixteen subbands as shown in Figure 1, where L_xL_y represent the approximation subbands and L_xH_y , H_xL_y and H_xH_y are the detail subbands, with $x=\overline{1,2}$.

The major advantage of multiwavelets over scalar wavelets is their ability to possess symmetry, orthogonality and higher order of approximation simultaneously, which is impossible for scalar wavelets. Furthermore, the multichannel structure of the multiwavelet transform is a closer approximation of the human visual system than what wavelets offer. In the case of unbalanced multiwavelets, the resulting approximation subbands carry different spectral content of the original image (both high- and low-frequencies), while for balanced multiwavelets, the approximation subbands contain similar spectral content of the original image [8]. This feature of unbalanced multiwavelets has the potential to increase the accuracy of the calculated disparity maps and to reduce the number of



(a) (b) (c)

Figure 2. Single level decomposition of Lena test image (a) Antonini 9/7 wavelet transform, (b) balanced bat01 multiwavelet transform and (c) unbalanced GHM multiwavelet transform.

erroneous matches compared to that of balanced multiwavelets.

Figures 2(a) to 2(c) give a visual comparison of the resulting subbands for the Antonini 9/7 scalar wavelet, as well as for the balanced bat01 and unbalanced GHM multiwavelets applied to the Lena test image. As it can be seen from Figure 2, multiwavelets generate four subbands instead of each subband that wavelets create. The resulting unbalanced multiwavelet subbands carry different spectral content of the original Lena test image, while the balanced multiwavelet subbands produce similar spectral content of the original image. More information about the generation of multiwavelets, their properties and their applications can be found in [4-7].

III. EVALUATION OF MULTIWAVELETS' FAMILY IN STEREO CORRESPONDENCE MATCHING

The proposed stereo correspondence matching system is based on multiwavelets and normalized cross correlation. Figures 3(a) and 3(b) show block diagrams of the proposed system for balanced and unbalanced multiwavelets respectively. A pair of stereo images is input to the stereo matching system. The images are first rectified to suppress the vertical displacement. A multiwavelet transform is then applied to each input stereo image. A number of different types and families of multiwavelets are evaluated. Since, the information in the approximation subbands is less sensitive to the shift variability of the multiwavelets, these subbands are used for correspondence matching purposes. In the case of balanced multiwavelets (Figure 3(a)), since their basebands contain similar spectral information, it is possible to use the shuffling technique proposed in [9] to rearrange the multiwavelet coefficients and generate a single low frequency subband. Figure 4 shows how four multiwavelet basebands are shuffled and a single baseband is formed. Figure 4(a) shows the four multiwavelet basebands with eight pixels (two from each baseband) highlighted and given a unique numeric label. Figure 4(b) shows the same set of pixels after shuffling, where coefficients corresponding to the same spatial locations in different basebands are placed together and one baseband is generated. Normalized cross correlation is then employed to find the best correspondence points between the two basebands of the stereo images and a disparity map is generated.

Figure 3(b) shows a block diagram of the unbalanced multiwavelet based stereo matching system. The shuffling technique works very well for balanced multiwavelets but it is not suitable for unbalanced multiwavelets due to their different spatio-frequency subband content. The unbalanced multiwavelets basebands contain both high and low frequency information with L_1L_1 (top left baseband) containing most of the image energy. For correspondence matching purposes, the same basebands from the two views are input to the normalized cross correlation block, generating four disparity maps as a result. As most of the image energy is concentrated in L_1L_1, its output disparity map is more reliable than the other three disparity maps

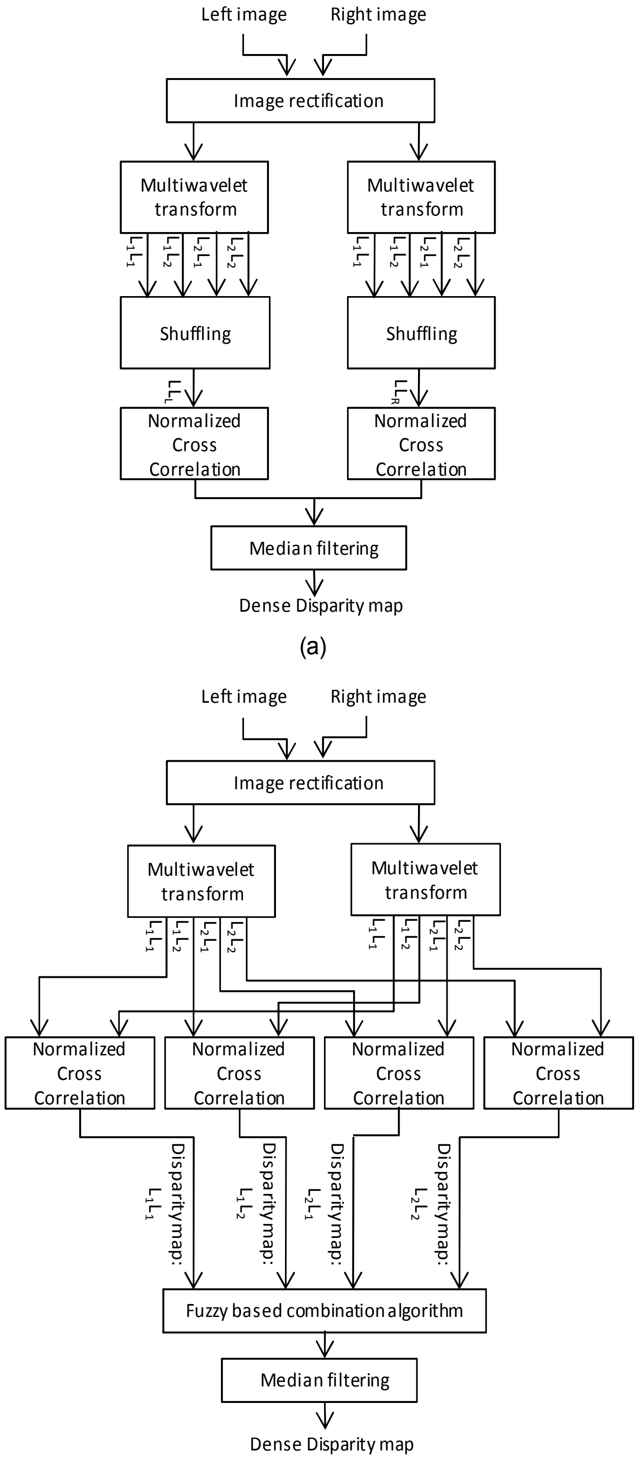


Figure 3. Block diagram of multiwavelet based stereo matching technique, (a) balanced- and (b) unbalanced-multiwavelets.

generated from other basebands, L_1L_2, L_2L_1, L_2L_2. Based on this property of unbalanced multiwavelets, a Fuzzy

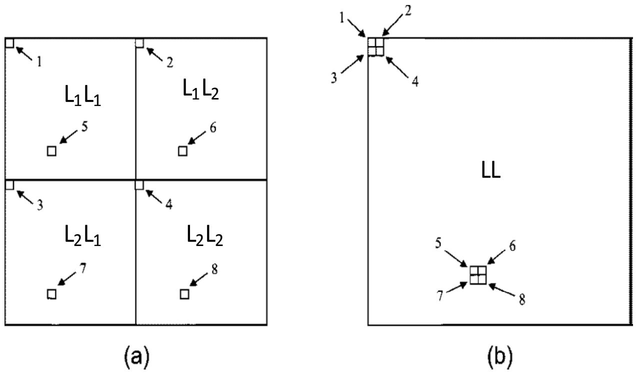


Figure 4. Shuffling method for multiwavelet baseband coefficients; selected pixels are numbered to indicate correspondence (a) before shuffling and (b) after shuffling.

algorithm is employed to combine the four disparity maps. This algorithm gives a higher weight to the disparity values of the L_1L_1 disparity map, while the disparity values of the other three disparity maps are used to refine the final disparity map. A median filter is then applied to further smooth the resulting disparity map.

IV. SIMULATION RESULTS

The performance of different types and families of multiwavelets in stereo correspondence has been evaluated using, 'Teddy' and 'Cones' stereo test images from the Middlebury stereo database [10]. Figure 5 shows the left image and the ground truth of these test images. The experimental results were generated using a number of multiwavelets, i.e. balanced versus unbalanced and symmetric-symmetric (SYM-SYM) versus symmetric-antisymmetric (SYM-ASYM) multiwavelets (listed in Table I). Table I shows the percentage of "bad pixels" at which the disparity error is larger than 1, for all regions (all). To give a visual comparison, the resulting disparity maps for balanced GHM and unbalanced BIGHM multiwavelets, applied to 'Cones' and 'Teddy' test images are shown in Figures 6(a) and 6(b), respectively. In these figures areas with intensity zero represent unreliable disparities. As can be seen from the results presented in Table I, generally unbalanced multiwavelets give better results compared to the balanced multiwavelets. The symmetric-symmetric multiwavelets seem to produce slightly better results compared to symmetric-antisymmetric multiwavelets (SA4). However, the symmetric-symmetric and symmetric-antisymmetric property of multiwavelets doesn't seem to have much effect on the resulting disparity map. From Figure 6, it is clear that the unbalanced multiwavelet based algorithm produces more accurate and smoother disparity maps compared to the balanced multiwavelet case. This can be explained by the fact that the approximation subbands of the unbalanced multiwavelet carry different spectral content of the input images, which enables the matching algorithm to generate more reliable matches.

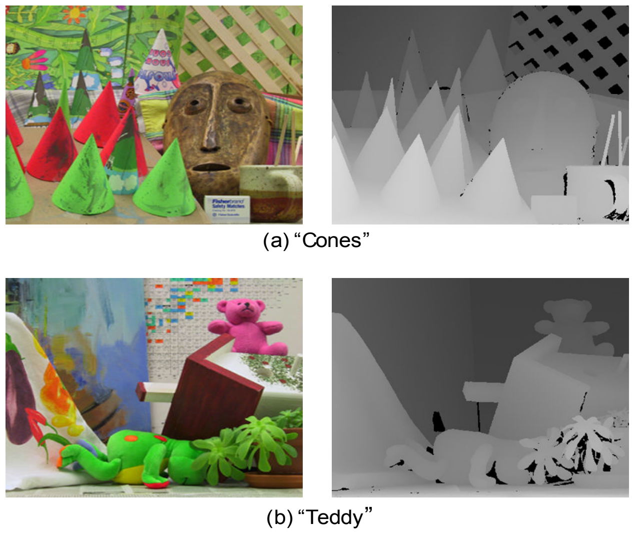


Figure 5. Left image and the ground truth of (a) 'Cones' and (b) 'Teddy'.

TABLE I. EVALUATION RESULTS OF DIFFERENT MULTIWAVELETS IN STEREO CORRESPONDENCE MATCHING.

'Teddy' (All)			
Balanced Multiwavelets		**Unbalanced Multiwavelets**	
CARDBAL2	9.84	BIH32S	8.92
CARDBAL 3	9.52	BIH52S(SYM-SYM)	8.91
BAT 01	10.37	BIH34N	8.92
BAT02	9.66	BIH54N (SYM-SYM)	8.99
GHM (SYM-SYM)	10.48	BIGHM	9.02
	9.84	SA4 (SYM-ASYM)	9.91
'Cones' (All)			
Balanced Multiwavelets		**Unbalanced Multiwavelets**	
CARDBAL2	8.84	BIH32S	8.75
CARDBAL 3	9.28	BIH52S(SYM-SYM)	8.85
BAT 01	9.34	BIH34N	8.74
BAT02	9.81	BIH54N (SYM-SYM)	8.91
GHM (SYM-SYM)	9.69	BIGHM	8.54
		SA4 (SYM-ASYM)	9.39

V. CONCLUSIONS

This paper has investigated the application of different types and families of multiwavelets in stereo correspondence matching. For this purpose, two correspondence matching algorithms were designed to deal with both balanced and unbalanced multiwavelets. In the case of balanced multiwavelets, due to the similar frequency content of the four multiwavelet low frequency subbands, they were shuffled to generate a single baseband and then normalized cross correlation was used to generate a disparity map. In the case of unbalanced multiwavelets, the four generated basebands and normalized cross correlation was used to generate four disparity maps. These maps were then combined using a Fuzzy algorithm to form a single disparity map. The results generated using Middlebury stereo test images show that unbalanced multiwavelets work better than balanced ones in stereo correspondence matching, while the symmetric-symmetric and symmetric-antisymmetric property of multiwavelets doesn't have a significant effect in reducing erroneous matches.

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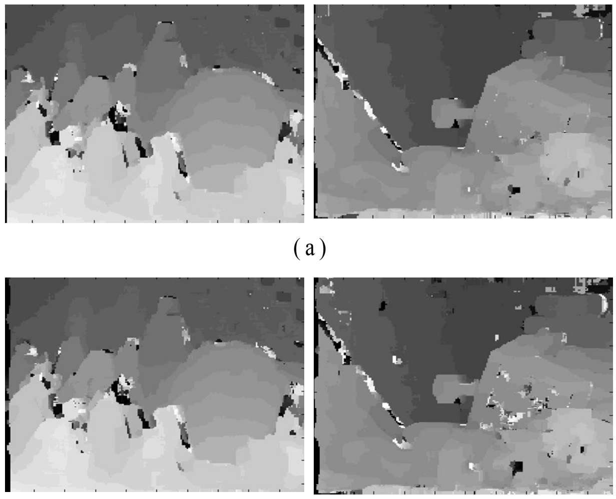

(a)

(b)

Figure 6. Disparity maps for 'Cones' and 'Teddy' stereo test image (a) unbalanced BIGHM and (b) balanced GHM multiwavelets.

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